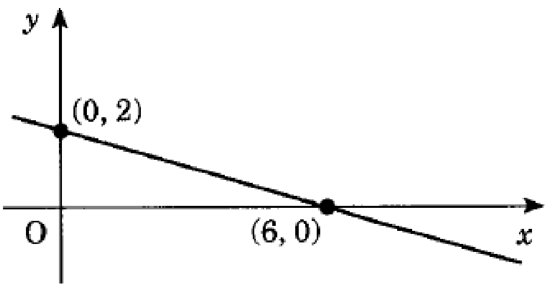
Differentiation

Basic Differentiation

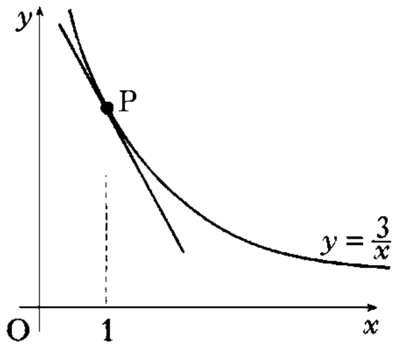
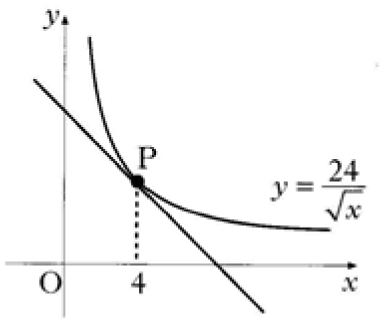
2003 P1	**5.** Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$.	**5**
2009 P2	**2.** Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$. (a) (i) Find $p(x)$ where $p(x) = f(g(x))$. (ii) Find $q(x)$ where $q(x) = g(f(x))$. (b) Solve $p'(x) = q'(x)$.	 **3** **3**
2015 P1	**7.** A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$. Find $f'(4)$.	**4**
2016 P1	**2.** Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$.	**3**
2017 P1	**8.** Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$.	**3**

Pre 2000 – Basic Differentiation

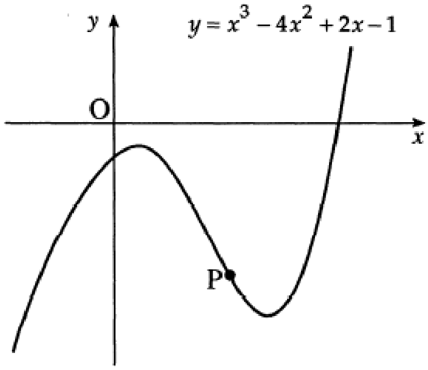
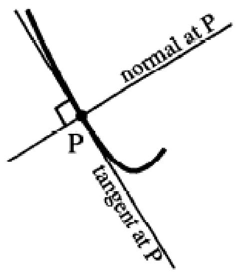
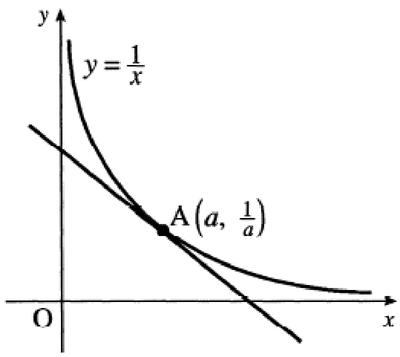
1	Differentiate $2\sqrt{x}(x+2)$ with respect to x.	4
2	Find $\frac{dy}{dx}$ where $y = \frac{4}{x^2} + x\sqrt{x}$.	4
3	Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1 + \frac{dy}{dx}\right) = 2y$.	3
4	If $y = x^2 - x$, show that $\frac{dy}{dx} = 1 + \frac{2y}{x}$.	3
5	Given $f(x) = 3x^2(2x-1)$ find $f'(-1)$.	3
6	Find $f'(4)$ where $f(x) = \frac{x-1}{\sqrt{x}}$.	5
7	If $f(x) = kx^3 + 5x - 1$ and $f'(1) = 14$, find the value of k.	3
8	Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$. (a) (i) Find $p(x)$ where $p(x) = f(g(x))$. (ii) Find $q(x)$ where $q(x) = g(f(x))$. **3** (b) Solve $p'(x) = q'(x)$. **3**	
9	The straight line shown in the diagram has equation $y = f(x)$. Determine $f'(x)$.  	2

10	A ball is thrown vertically upwards. After t seconds its height is h metres, where $h = 1.2 + 19.6t - 4.9t^2$. (*a*) Find the speed of the ball after 1 second. (*b*) For how many seconds is the ball travelling upwards?	3 2
11	A ball is thrown vertically upwards. The height h metres of the ball t seconds after it is thrown, is given by the formula $h = 20t - 5t^2$. (*a*) Find the speed of the ball when it is thrown (i.e. the rate of change of height with respect to time of the ball when it is thrown). (*b*) Find the speed of the ball after 2 seconds. Explain your answer in terms of the movement of the ball.	3 2

Equations of Tangents

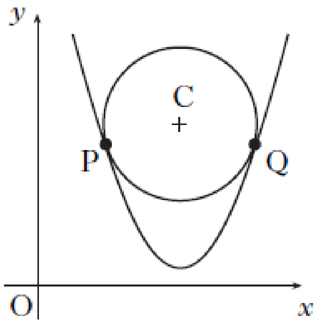
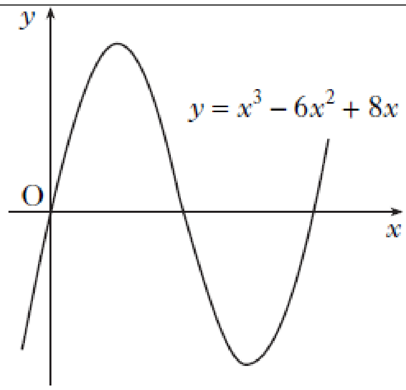
2001 P2	**2.** A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $x > 0$. Find the equation of the tangent at the point where $x = 4$.	**6**
6.(JAN) 02 P1	Find the equation of the tangent to the curve with equation $y = \frac{3}{x}$ at the point P where $x = 1$.  	**5**
2005 P2	**6.** The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$. Find the equation of the tangent at P, where $x = 4$.  	**6**
2014 P2	**2.** A curve has equation $y = x^4 - 2x^3 + 5$. Find the equation of the tangent to this curve at the point where $x = 2$.	**4**
2015 EP P1	**1.** The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$. Find the equation of the tangent to this curve at P.	**3**
2015 P1	**2.** Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$.	**4**

Pre 2000 – Equation of Tangent

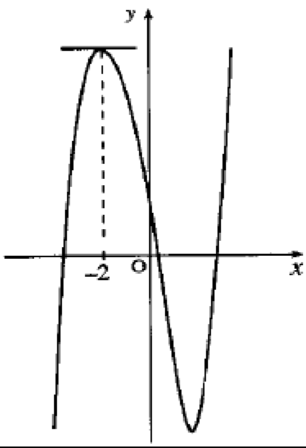
1	Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$.	4
2	Find the equation of the tangent to the curve $y = 3x^2 + 2$ at the point where $x = 1$.	4
3	Find the equation of the tangent to the curve $y = 4x^3 - 2$ at the point where x = –1.	4
4	The point P(–1, 7) lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P.	4
5	(*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$. 	**(5)**
	(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P. Find the angle which the normal at P makes with the positive direction of the *x*-axis. 	**(2)**
11	(*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a, \frac{1}{a}\right)$ has been drawn. Find the gradient of this tangent. 	**(4)**
	(*b*) Hence show that the equation of this tangent is $x + a^2y = 2a$.	**(2)**
	(*c*) This tangent cuts the *y*-axis at B and the *x*-axis at C.	
	(i) Calculate the area of triangle OBC	**(3)**
	(ii) Comment on your answer to *c*(i).	**(1)**

Tangent Problems

2002 P1	**4.** Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis.	**4**

2004 P2	**5.** The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$. (*a*) Find the value of x for which the gradient of the tangent at P is 12. (*b*) Hence find the equation of the tangent at P.	 **5** **2**
2007 P2	**5.** A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.  (*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. (*b*) Find the coordinates of P. (*c*) Find the coordinates of C, the centre of the circle.	 **5** **2** **2**
2008 SP2 P2	**3.** The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$. (*a*) Find the value of x for which the gradient of the tangent at P is 12. (*b*) Hence find the equation of the tangent at P.	 **5** **2**
2008 P1	**22.** The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.  (*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. (*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A.	 **5** **2**

Pre 2000 Questions – Tangent Problems

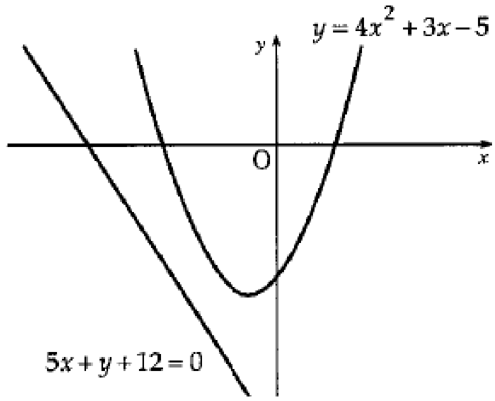
1	Find the x-coordinate of each of the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent is parallel to the x-axis.	**4**
2	The diagram shows a sketch of the curve $y = x^3 + kx^2 - 8x + 3$. The tangent to the curve at $x = -2$ is parallel to the x-axis. Find the value of k. 	**4**

3

The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.

A tangent to the parabola is drawn parallel to the given straight line.

Find the x-coordinate of the point of contact of this tangent.



5

4

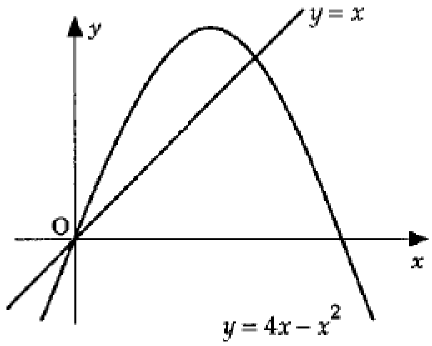
Calculate, to the nearest degree, the angle between the x-axis and the tangent to the curve with equation $y = x^3 - 4x - 5$ at the point where $x = 2$.

4

5

Find the gradient of the tangent to the parabola $y = 4x - x^2$ at (0,0).

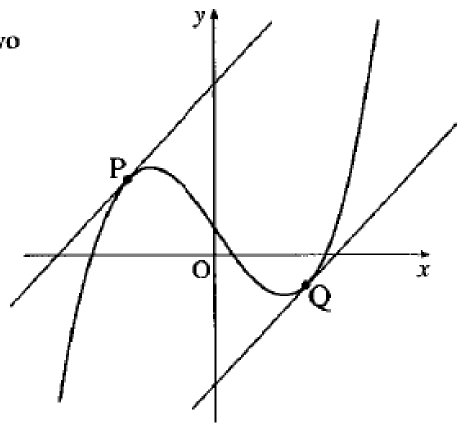
Hence calculate the size of the angle between the line $y = x$ and this tangent.



6

6

The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.

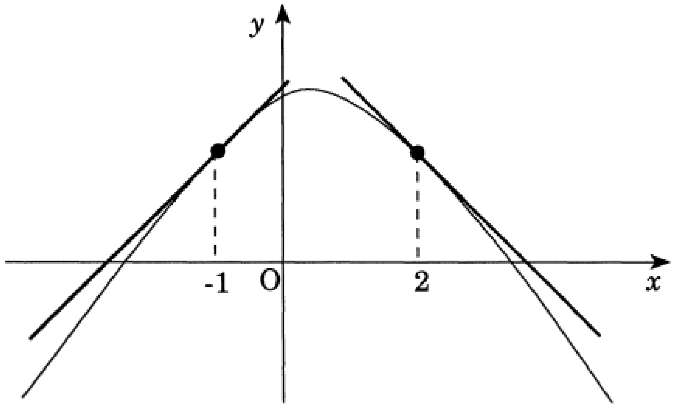


(*a*) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have gradient 3. **6**

(*b*) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **6**

7

The parabola $y = ax^2 + bx + c$ crosses the y-axis at (0, 3) and has two tangents drawn, as shown in the diagram.

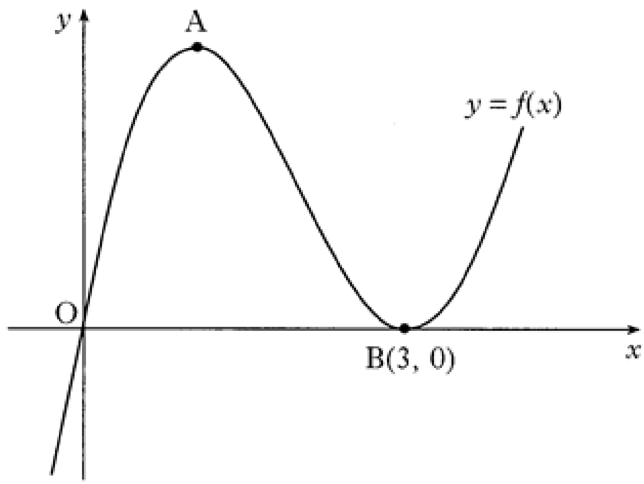


The tangent at $x = -1$ makes an angle of 45° with the positive direction of the x-axis and the tangent at $x = 2$ makes an angle of 135° with the positive direction of the x-axis.
Find the values of a, b and c. **(8)**

Stationary Points

2000 P1

A2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(a) Find the coordinates of the turning point at A. **4**

(b) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(c) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

2007 P1

9. A function f is defined by the formula $f(x) = 3x - x^3$.

(a) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

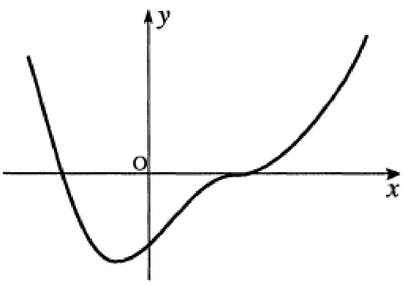
(b) Find the coordinates of the stationary points of the function and determine their nature. **7**

(c) Sketch the graph of $y = f(x)$. **1**

2008 SP1 P1	**22.** (*a*) Find the stationary points on the curve with equation $y = x^3 - 9x^2 + 24x - 20$ and justify their nature. **7** (*b*) (i) Show that $(x-2)^2(x-5) = x^3 - 9x^2 + 24x - 20$. (ii) Hence sketch the graph of $y = x^3 - 9x^2 + 24x - 20$. **4**
2008 SP2 P1	**21.** (*a*) Find the stationary points on the curve with equation $y = x^3 + 3x^2 - 9x + 5$ and justify their nature. **7** (*b*) The curve passes through the point $(-5, 0)$. Sketch the curve. **2**
2009 P2	**1.** Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**
2011 P1	**22.** A function f is defined on the set of real numbers by $f(x) = (x-2)(x^2+1)$. (*a*) Find where the graph of $y = f(x)$ cuts: (i) the x-axis; (ii) the y-axis. **2** (*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8** (*c*) On separate diagrams sketch the graphs of: (i) $y = f(x)$; (ii) $y = -f(x)$. **3**
2014 P1	**21.** A curve has equation $y = 3x^2 - x^3$. (*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6** (*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

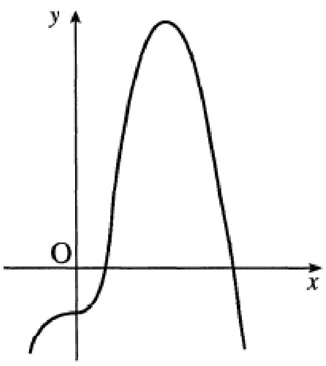
Pre 2000 – Stationary Points

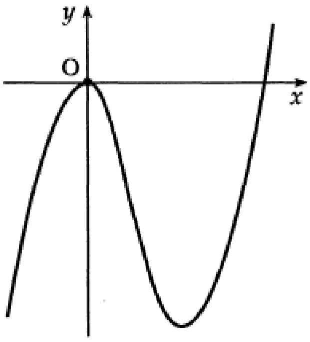
1 The function f, whose incomplete graph is shown in the diagram, is defined by
$f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature.



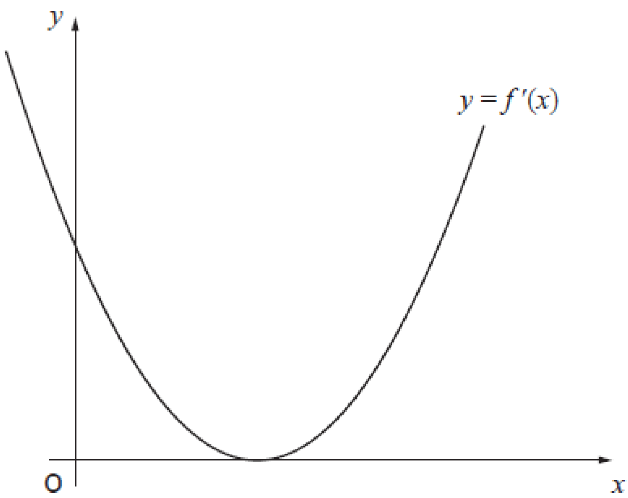
(8)

2 A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.
(*a*) Find the coordinates of the stationary points. **(6)**
(*b*) Determine the nature of the stationary points. **(2)**



3	A curve has equation $y = x^4 - 4x^3 + 3$. (a) Find algebraically the coordinates of the stationary points. **(6)** (b) Determine the nature of the stationary points. **(2)**
4	(a) The diagram shows a part of the curve with equation $y = 2x^2(x-3)$. Find the coordinates of the stationary points on the graph and determine their nature. **(5)**  (b) State the range of values of k for which $y = k$ intersects the graph in three distinct points. **(2)**
5	A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbf{R}$. (a) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **(3)** (b) Find the stationary points of this curve $y = f(x)$ and determine their nature. **(7)** (c) Sketch the curve $y = f(x)$. **(2)**

Increasing/ Decreasing Functions

2004 P1	**8.** (a) Write $x^2 - 10x + 27$ in the form $(x+b)^2 + c$. **2** (b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**
2015 SP P1	11. The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.  (a) Explain why the function $f(x)$ is never decreasing. 1 (b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$. 2
2016 P1	9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. 4 (b) Hence determine the range of values of x for which the function f is strictly increasing. 2

2017 P2	4. (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. (b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$. (c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x.	3 2 2

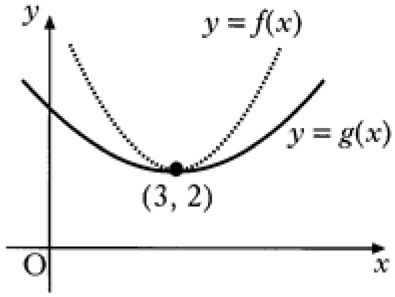
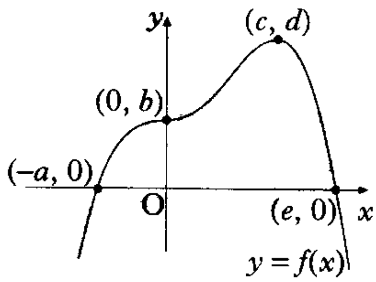
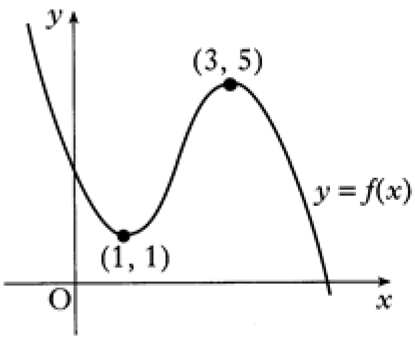
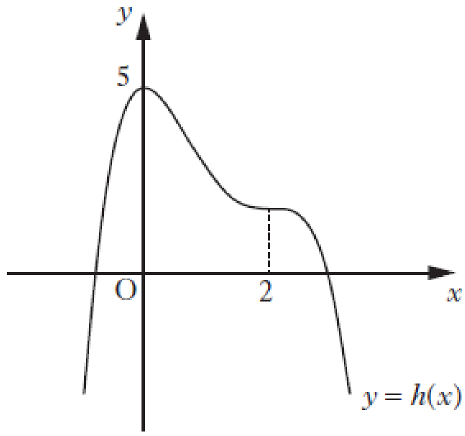
Pre 2000 – Increasing/Decreasing Functions

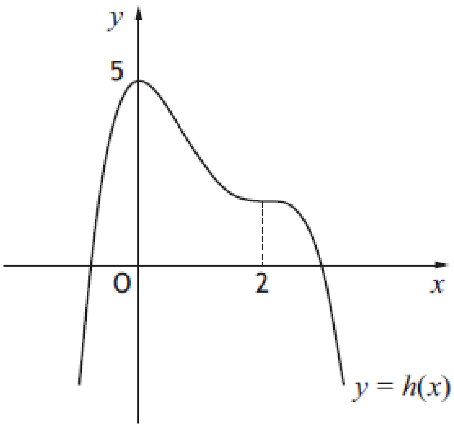
1	Find algebraically the values of x for which the function $f(x)=2x^3-3x^2-36x$ is increasing.	4
2	The point P(–2, b) lies on the graph of the function $f(x)=3x^3-x^2-7x+4$. (*a*) Find the value of b. (*b*) Prove that this function is increasing at P.	1 3
3	For what values of x is the function $f(x)=\frac{1}{3}x^3-2x^2-5x-4$ increasing?	5

Max/Min values within a Closed Intervals

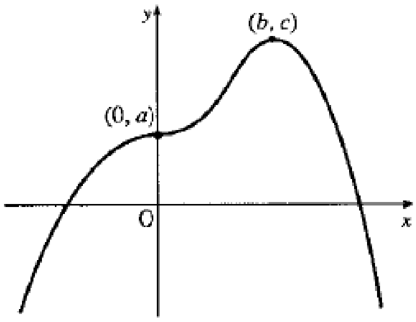
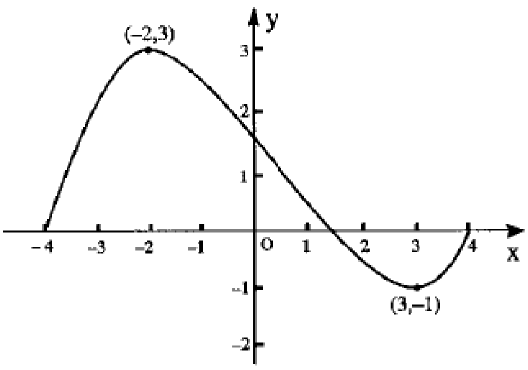
2012 P2	3. A function f is defined on the domain $0\le x\le 3$ by $f(x)=x^3-2x^2-4x+6$. Determine the maximum and minimum values of f.	7
2015 OLD P1	22. The function $f(x)=\frac{4}{x^2}+x$ is defined on the domain $x>0$, $x\in\mathbb{R}$, the set of real numbers. Find the maximum and minimum values of $f(x)$ on the closed interval $1\le x\le 4$.	6
2017 P2	7. (a) Find the x-coordinate of the stationary point on the curve with equation $y=6x-2\sqrt{x^3}$. (b) Hence, determine the greatest and least values of y in the interval $1\le x\le 9$.	4 3

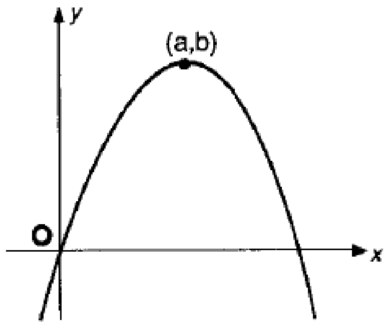
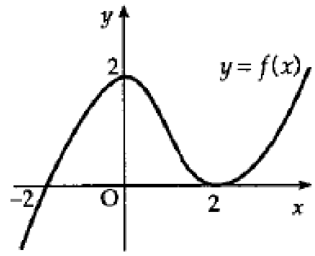
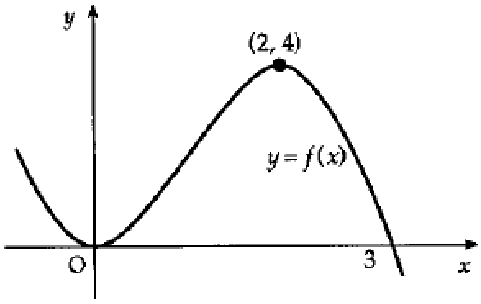
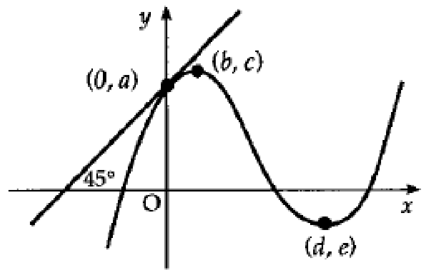
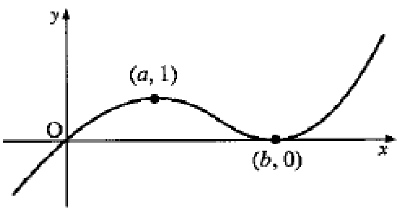
Graphs of y = f ‘(x)

2001 P1	**9.** The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at (3, 2). Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$. 	**2**
2002 P1	**6.** The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown. There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d). Sketch the graph of the derived function f'. 	**3**
2004 P2	**7.** The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5). Sketch the graph of $y = f'(x)$. 	**3**
2012 P2	**4.** The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.  On separate diagrams sketch the graphs of: (*a*) $y = h'(x)$; (*b*) $y = 2 - h'(x)$.	3 3

2015 EP P1	**8.** The diagram below shows the graph of a quartic $y=h(x)$, with stationary points at $x=0$ and $x=2$.  On separate diagrams sketch the graphs of: (a) $y=2-h(x)$. **3** (b) $y=h'(x)$. **3**

Pre 2000 – Graphs of y = f '(x)

1	The diagram shows a sketch of part of the graph of $y=f(x)$. The graph of has a point of inflection at $(0, a)$ and a maximum turning point at (b, c). (a) Make a copy of this diagram and on it sketch the graph of $y=g(x)$ where $g(x)=f(x)+1$. (b) On a separate diagram sketch the graph of $y=f'(x)$. (c) Describe how the graph of $y=g'(x)$ is related to the graph of $y=f'(x)$.  	2 2 1
2	A sketch of a cubic function, f, with domain $-4 \leq x \leq 4$, is shown in the diagram below. Sketch the graph of the derived function, f', for the same domain.  	3

3	The line with equation $y = x$ is a tangent at the origin to the parabola with equation $y = f(x)$. The parabola has a maximum turning point at (a, b). Sketch the graph of $y = f'(x)$.  	4
4	The diagram shows the graph of $y = f(x)$, where $-2 \le x \le 3$. On separate diagrams, sketch the graphs of (*a*) $y = -f(x)$; (*b*) $y = f'(x)$.  	2 3
5	The diagram shows a sketch of a cubic function f with stationary points at (0, 0) and (2, 4). Sketch the graph of the derived function f'.  	3
6	The diagram shows the graph of a cubic function with a maximum at (b, c) and a minimum at (d, e). The tangent at $(0, a)$ is inclined at 45° to the x-axis. (*a*) State the values of $f'(b)$, $f'(d)$ and $f'(0)$. (*b*) Sketch the graph of the the derived function f'.  	2 2
7	A sketch of the graph of the cubic function f is shown. It passes through the origin, has a maximum turning point at $(a, 1)$ and a minimum turning point at $(b, 0)$. (*a*) Make a copy of this diagram and on it sketch the graph of $y = 2 - f(x)$, indicating the coordinates of the turning points. (*b*) On a separate diagram sketch the graph of $y = f'(x)$. (*c*) The tangent to $y = f(x)$ at the origin has equation $y = \frac{1}{2}x$. Use this information to write down the coordinates of a point on the graph of $y = f'(x)$.  	3 2 1